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THE ANTIPLANE PROBLEM OF THE MOTION OF A POINT SHEAR LOAD OVER A TWO-LAYER ELASTIC BASE AT A CONSTANT VELOCITY†

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The problem of modelling the motion of a force disturbance in an elastic medium that is heterogeneous over its depth is investigated. It is in an antiplane formulation in a moving system of coordinates that all possible versions of the ratio of the velocity of motion of the surface point shear load to the velocities of the shear waves in the layers of the two-layer elastic base are examined. Cases of a subsonic regime (SBR) in the upper and lower layers, of a supersonic regime (SPR) in the upper layer and an SBR in the lower layer, and of an SBR in the upper layer and an SPR in the lower layer are studied using the Fourier transform and the theory of residues. The last two cases are extremely interesting from the mathematical point of view, as here, on the boundary between the layers, the solutions of elliptic and hyperbolic equations meet, and previously unknown features arise in the displacements that, it seems, should also occur in the solution of the corresponding plane problem. The case of an SPR in the upper and lower layers is investigated using a special method for successive allowance for the incident, reflected and refracted shock wave fronts. In all cases, expressions are obtained for the displacements in the layers, and their characteristic features are investigated. © 2001 Elsevier Science Ltd. All rights reserved.

1. FORMULATION OF THE PROBLEM

Consider a two-layer elastic base. The upper layer (1) has a height h, and the lower layer (2) has a height H. The lower layer is restrained by the base, and complete adhesion is achieved between the layers. We will introduce the coordinate system $Oxyz$, as shown in Fig. 1, moving in the direction of the x axis at a velocity v. We will assume that a point shear load T acts on the base at the point $x = 0, y = h$, referred to unit length and independent of z.

Suppose $c_1 = \sqrt{G_1/\rho_1}$ and $c_2 = \sqrt{G_2/\rho_2}$ are the velocities of propagation of shear waves in the layers, where G_1, G_2 and ρ_1, ρ_2 are their shear moduli and the densities of the materials. When $v < c_1$ and $v < c_2$, to solve the problem we will have the equations

$$\gamma^2 \frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} = 0, \quad \delta^2 \frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} = 0 \tag{1.1}$$

$$\gamma = \sqrt{1 - \frac{v^2}{c_1^2}}, \quad \delta = \sqrt{1 - \frac{v^2}{c_2^2}}$$

$$\tau_{xz}^{(j)} = G_j \frac{\partial w_j}{\partial x}, \quad \tau_{yz}^{(j)} = G_j \frac{\partial w_j}{\partial y} \quad (j = 1, 2)$$

When $v > c_1$ and $v < c_2$, in the first equation of (1.1) it is necessary to replace γ by $i\gamma_*$, and when $v < c_1$ and $v > c_2$ in the second equation of (1.1) we must replace δ by $i\delta_*$, where

$$\gamma_* = \sqrt{\frac{v^2}{c_1^2} - 1}, \quad \delta_* = \sqrt{\frac{v^2}{c_2^2} - 1} \tag{1.2}$$

When $v > c_1$ and $v > c_2$, these replacements in both equations of (1.1) must be carried out simultaneously.

The boundary conditions of the problem have the form

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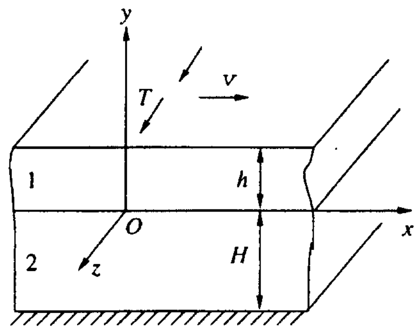

Fig. 1

$$\begin{aligned} y = -H:&\quad w_2 = 0 \\ y = 0:&\quad w_1 = w_2, \quad \tau_{yz}^{(1)} = \tau_{yz}^{(2)} \\ y = h:&\quad \tau_{yz}^{(1)} = T\delta(x) \end{aligned} \tag{1.3}$$

where $\delta(x)$ is the delta function.

2. THE CASE OF AN SBR IN THE UPPER AND LOWER LAYERS

Suppose $v < c_1$ and $v < c_2$, where both Eqs (1.1) are equations of elliptic type. We will seek a solution of the problem in the form of Fourier integrals [1]

$$w_j = \frac{1}{2\pi}\int_{-\infty}^{\infty} W_j(\alpha, y)e^{-i\alpha x}d\alpha \tag{2.1}$$

From Eqs (1.1) we have

$$\begin{aligned} W_1(\alpha, y) &= A_1(\alpha)\operatorname{sh}(\alpha\gamma y) + B_1(\alpha)\operatorname{ch}(\alpha\gamma y) \\ W_2(\alpha, y) &= A_2(\alpha)\operatorname{sh}(\alpha\delta y) + B_2(\alpha)\operatorname{ch}(\alpha\delta y) \end{aligned} \tag{2.2}$$

Satisfying boundary conditions (1.3), we find $A_j(\alpha)$ and $B_j(\alpha)$ and obtain

$$w_j = \frac{T}{2\pi G_1\gamma}\int_{-\infty}^{\infty}\frac{\Delta_j(\alpha, y)}{\alpha\Delta(\alpha)}e^{-i\alpha x}d\alpha \tag{2.3}$$

$$\Delta_1(\alpha, y) = k\operatorname{sh}(\alpha\gamma y)\operatorname{ch}(\alpha\delta H) + \operatorname{sh}(\alpha\delta H)\operatorname{ch}(\alpha\gamma y)$$

$$\Delta_2(\alpha, y) = \operatorname{sh}(\alpha\delta y)\operatorname{ch}(\alpha\delta H) + \operatorname{sh}(\alpha\delta H)\operatorname{ch}(\alpha\delta y)$$

$$\Delta(\alpha) = k\operatorname{ch}(\alpha\gamma h)\operatorname{ch}(\alpha\delta H) + \operatorname{sh}(\alpha\gamma h)\operatorname{sh}(\alpha\delta H), \quad k = G_2\delta/(G_1\gamma)$$

We will expand the integrals in (2.3) in residues. It can be shown that the roots of the transcendental equation $\Delta(\alpha) = 0$ in the complex plane $z = \alpha\gamma h$ are single imaginary roots. They are given in Table 1 for certain values of the parameters k and $l = \delta H(\gamma h)$, where $r = 0, 1, \ldots$

Denoting by $\alpha_s = ia_s$ $(s = 1, 2, \ldots)$ the roots of the equation $\Delta(\alpha) = 0$ lying in the upper half-plane, we obtain

$$\begin{aligned} w_1 &= \frac{T}{G_1\gamma}\sum_{s=1}^{\infty}\frac{k\sin(\alpha_s\gamma y)\cos(a_s\delta H) + \sin(\alpha_s\delta H)\cos(a_s\gamma y)}{a_sD(a_s)}e^{-a_s|x|} \\ w_2 &= \frac{T}{G_1\gamma}\sum_{s=1}^{\infty}\frac{\sin(\alpha_s\delta y)\cos(a_s\delta H) + \sin(\alpha_s\delta H)\cos(a_s\delta y)}{a_sD(a_s)}e^{-a_s|x|} \\ D(a_s) &= (k\gamma h + \delta H)\sin(\alpha_s\gamma h)\cos(a_s\delta H) + (k\delta H + \gamma h)\sin(\alpha_s\delta H)\cos(a_s\gamma h) \end{aligned} \tag{2.4}$$

Table 1

k	$l = 2$	$l = 1$	$l = 1/2$
2	$\pm i(0.6154797 + \pi r)$	$\pm i(0.9553166 + \pi r)$	$\pm i(1.2309594 + \pi r)$
1	$\pm i(\pi/6 + \pi r)$	$\pm i(\pi/4 + \pi r)$	$\pm i(\pi/3 + \pi r)$
1/2	$\pm i(0.4205343 + \pi r)$	$\pm i(0.6154797 + \pi r)$	$\pm i(0.8410687 + \pi r)$

Since $a_s \sim \pi s$ when $s \to \infty$, it is not difficult to establish that the series for w_1 converges to a continuous function at all $0 \leqslant y \leqslant h$ and $|x| < \infty$, excluding, when $y = h$, the neighbourhood of the point $x = 0$, where it behaves, apart from a factor, as $-\ln|x|$. The series for w_2 converges to continuous function for all $-H \leqslant y \leqslant 0$ and $|x| < \infty$.

3. THE CASE OF AN SPR IN THE UPPER LAYER AND AN SBR IN THE LOWER LAYER

Suppose $v > c_1$ and $v < c_2$, where Eqs (1.1) are equations of hyperbolic and elliptic type respectively. In formulae (2.3) it is necessary to replace γ by $i\gamma_*$, and k by mi^{-1}, where $m = G_2\delta/(G_1\gamma_*)$. As a result we will have

$$w_j = \frac{T}{2\pi G_1\gamma_*}\int_\Gamma \frac{\Delta_j(\alpha, y)}{\alpha\Delta(\alpha)}e^{-i\alpha x}d\alpha \tag{3.1}$$

$$\Delta_1(\alpha, y) = m\sin(\alpha\gamma_* y)\operatorname{ch}(\alpha\delta H) + \operatorname{sh}(\alpha\delta H)\cos(\alpha\gamma_* y)$$

$$\Delta_2(\alpha, y) = \operatorname{sh}(\alpha\delta y)\operatorname{ch}(\alpha\delta H) + \operatorname{sh}(\alpha\delta H)\operatorname{ch}(\alpha\delta y)$$

$$\Delta(\alpha) = m\cos(\alpha\gamma_* h)\operatorname{ch}(\alpha\delta H) - \sin(\alpha\gamma_* h)\operatorname{sh}(\alpha\delta H)$$

We will expand the integral in (3.1) in residues. It can be shown that the transcendental equation $\Delta(\alpha) = 0$ in the complex plane $z = \alpha\gamma_* h$ will now have single roots that are both imaginary and real. They are presented in Table 2 (the imaginary roots) and in Table 3 (the real roots) for certain values of the parameters m and $n = \delta H/(\gamma_* h)$, where $r = 0, 1, \ldots$ The integration contour Γ in (3.1) will be selected so that it proceeds along the real axis, bypassing the real poles from above when $x < 0$ and from below when $x > 0$.

Denoting by $\alpha_s = ia_s$ ($s = 1, 2, \ldots$) the imaginary roots lying in the upper half-plane, and by β_s ($s = 1, 2, \ldots$) the real roots in the right-hand half-plane, we obtain

Table 2

m	$n = 2$	$n = 1$	$n = 1/2$
2	$\pm i(0.9639096 + \pi r/2)$	$\pm i(2.0205285 + \pi r)$	$\pm i(4.0684198 + 2\pi r)$
1	$\pm i(1.1256602 + \pi r/2)$	$\pm i(2.3470456 + \pi r)$	$\pm i(4.7122275 + 2\pi r)$
1/2	$\pm i(1.3084609 + \pi r/2)$	$\pm i(2.6741290 + \pi r)$	$\pm i(5.3558443 + 2\pi r)$

Table 3

k	$n = 2$	$n = 1$	$n = 1/2$
2	$\pm(1.1162877 + \pi r)$	$\pm(1.1786902 + \pi r)$	$\pm(1.2934123 + \pi r)$
1	$\pm(0.8226173 + \pi r)$	$\pm(0.9375520 + \pi r)$	$\pm(1.1051875 + \pi r)$
1/2	$\pm(0.5559512 + \pi r)$	$\pm(0.6941913 + \pi r)$	$\pm(0.8797559 + \pi r)$

$$w_1 = \frac{T}{G_1\gamma_*}\left\{\sum_{s=1}^{\infty}\frac{m\,\mathrm{sh}(a_s\gamma_* y)\cos(a_s\delta H)+\sin(a_s\delta H)\mathrm{ch}(a_s\gamma_* y)}{a_s D(a_s)}e^{-a_s|x|} - \right.$$
$$\left. -\sum_{s=1}^{\infty}\frac{\Delta_1(\beta_s, y)}{\beta_s D_*(\beta_s)}\sin(\beta_s|x|)\right\}$$
$$w_2 = \frac{T}{G_1\gamma_*}\left\{\sum_{s=1}^{\infty}\frac{\sin(a_s\delta y)\cos(a_s\delta H)+\sin(a_s\delta H)\cos(a_s\delta y)}{a_s D(a_s)}e^{-a_s|x|} - \right. \tag{3.2}$$
$$\left. -\sum_{s=1}^{\infty}\frac{\Delta_2(\beta_s, y)}{\beta_s D_*(\beta_s)}\sin(\beta_s|x|)\right\}$$
$$D(a_s) = -(m\gamma_* h+\delta H)\mathrm{sh}(a_s\gamma_* h)\cos(a_s\delta H)+(m\delta H-\gamma_* h)\mathrm{ch}(a_s\gamma_* h)\sin(a_s\delta H)$$
$$D_*(\beta_s) = -(m\gamma_* h+\delta H)\sin(\beta_s\gamma_* h)\mathrm{ch}(\beta_s\delta H)+(m\delta H-\gamma_* h)\cos(\beta_s\gamma_* h)\mathrm{sh}(\beta_s\delta H)$$

Since $a_s \sim \pi s/n$ as $s \to \infty$, it is not difficult to show that the first series in the expression for w_1 converges to a continuous function for all $0 \leqslant y \leqslant h$ and $|x| < \infty$, with the exception, when $y = h$, of the neighbourhood of the point $x = 0$, where it behaves, apart from a factor, as $-\ln|x|$. With regard to the second series in w_1, taking into account that $\beta_s \sim \pi s$ when $s \to \infty$, it can shown that its principal part comprises the function

$$\frac{1}{2(m\delta H-\gamma_* h)}\left\{m\left[\ln\left|2\cos\left(\frac{t_- - 2r}{2}\right)\right| - \ln\left|2\cos\left(\frac{t_+ - 2r}{2}\right)\right|\right] + \right.$$
$$\left. + \frac{t_- + t_+ - 4r}{2}\right\} \quad \left(t_\pm = \frac{1}{h}\left(\frac{|x|}{\gamma_*} \pm y\right)\right) \tag{3.3}$$

where $r = 0, 1, \ldots$, and $-1 < t_- < 1$ and $0 \leqslant t_+ < 1$ when $r = 0$, and $(2r - 1) < t_\pm < (2r + 1)$ when $r \geqslant 1$. Thus, the second series in w_1 has discontinuities of the first kind of the sgn type and discontinuities of the second kind of the logarithmic type on the lines

$$|x| = \pm\gamma_* y + (2r \mp 1)\gamma_* h \quad (r = 0, 1, \ldots, \quad 0 < y < h) \tag{3.4}$$

When $y = h$, the principal part of the second series in w_1 comprises the function

$$\frac{t-2r-1}{2(m\delta H-\gamma_* h)} \quad \left(t = \frac{|x|}{\gamma_* h}\right) \tag{3.5}$$

where $r = 0, 1, \ldots$ and $2r < t < 2r + 2$, and when $y = 0$ it comprises the function

$$\frac{t-2r}{2(m\delta H-\gamma_* h)} \tag{3.6}$$

where $r = 0, 1, \ldots$, and here $0 \leqslant t < 1$ when $r = 0$, and $2r - 1 < t < 2r + 1$ when $r \geqslant 1$. Thus, when $y = h$, the second series in w_1 has discontinuities of the first kind of the sgn type at the points $|x| = (2r + 2)\gamma_* h$ $(r = 0, 1, \ldots)$ and a break at the point $x = 0$, and when $y = 0$ it has discontinuities of the first kind of the sgn type at the points $|x| = (2r + 1)\gamma_* h$ $(r = 0, 1, \ldots)$ and a break at the point $x = 0$. Discontinuities of the second kind of the logarithmic type in w_1 do not reach the surfaces $y = h$ and $y = 0$.

Results (3.3)–(3.6) can be arrived at on the basis of the relationships in [2, (1.441 (1, 3, 4))].

The first series in the expression for w_2 converges to a continuous function for all $-H \leqslant y \leqslant 0$ and $|x| < \infty$. The second series in w_2 converges to a continuous function at all $-H \leqslant y < 0$ and $|x| < \infty$; when $y = 0$ it chiefly comprises the function (3.6), i.e. it obviously behaves in the same way as the second series in w_1 when $y = 0$.

4. THE CASE OF AN SBR IN THE UPPER LAYER AND AN SPR IN THE LOWER LAYER

Suppose $\upsilon < c_1$ and $\upsilon > c_2$, where Eqs (1.1) are equations of elliptic and hyperbolic type respectively. In formulae (2.3) it is necessary to replace δ by $i\delta_*$ and k by $-pi^{-1}$, where $p = G_2\delta_*/(G_1\gamma)$. As a result we will have

$$w_j = \frac{T}{2\pi G_1\gamma}\int_\Gamma \frac{\Delta_j(\alpha, y)}{\alpha\Delta(\alpha)} e^{-i\alpha x} d\alpha \tag{4.1}$$

$$\Delta_1(\alpha, y) = p\,\mathrm{sh}(\alpha\gamma y)\cos(\alpha\delta_* H) + \sin(\alpha\delta_* H)\,\mathrm{ch}(\alpha\gamma y)$$

$$\Delta_2(\alpha, y) = \sin(\alpha\delta_* y)\cos(\alpha\delta_* H) + \sin(\alpha\delta_* H)\cos(\alpha\delta_* y)$$

$$\Delta(\alpha) = p\,\mathrm{ch}(\alpha\gamma h)\cos(\alpha\delta_* H) + \mathrm{sh}(\alpha\gamma h)\sin(\alpha\delta_* H)$$

We expand the integrals in (4.1) in residues. It can be shown that the transcendental equation $\Delta(\alpha) = 0$ in the complex plane $z = \alpha\gamma h$ will again have single roots that are both imaginary and real. They are presented in Table 3 (the imaginary roots, $m = p$, $n = q$, before all values it is necessary to add the square root of -1, i) and in Table 2 (the real roots, $m = p$, $n = q$, before all values it is necessary to remove the square root of -1, i.e. i) for certain values of the parameters p and $q = \delta_* H/(\gamma h)$, where $r = 0, 1, \ldots$ We will again select the contour of integration Γ in (4.1) so that it proceeds along the real axis, bypassing the real poles from above when $x < 0$ and from below when $x > 0$.

Denoting by $\alpha_s = ia_s$ ($s = 1, 2, \ldots$) the real roots lying in the upper half-plane, and by β_s ($s = 1, 2, \ldots$) the real roots in the right-hand half-plane, as above, we obtain

$$w_1 = \frac{T}{G_1\gamma}\left\{\sum_{s=1}^{\infty} \frac{p\sin(a_s\gamma y)\,\mathrm{ch}(a_s\delta_* H) + \mathrm{sh}(a_s\delta_* H)\cos(a_s\gamma y)}{a_s D(a_s)} e^{-a_s|x|} - \right.$$
$$\left. -\sum_{s=1}^{\infty} \frac{\Delta_1(\beta_s, y)}{\beta_s D_*(\beta_s)}\sin(\beta_s|x|)\right\} \tag{4.2}$$
$$w_2 = \frac{T}{G_1\gamma}\left\{\sum_{s=1}^{\infty} \frac{\mathrm{sh}(\alpha_s\delta_* y)\,\mathrm{ch}(a_s\delta_* H) + \mathrm{sh}(a_s\delta_* H)\,\mathrm{ch}(a_s\delta_* y)}{a_s D(a_s)} e^{-a_s|x|} - \right.$$
$$\left. -\sum_{s=1}^{\infty} \frac{\Delta_2(\beta_s, y)}{\beta_s D_*(\beta_s)}\sin(\beta_s|x|)\right\}$$

$$D(a_s) = (p\gamma h + \delta_* H)\sin(\alpha_s\gamma h)\,\mathrm{ch}(a_s\delta_* H) - (p\delta_* H - \gamma h)\cos(a_s\gamma h)\,\mathrm{sh}(a_s\delta_* H)$$
$$D_*(\beta_s) = (p\gamma h + \delta_* H)\,\mathrm{sh}(\beta_s\gamma h)\cos(\beta_s\delta_* H) - (p\delta_* H - \gamma h)\,\mathrm{ch}(\beta_s\gamma h)\sin(\beta_s\delta_* H)$$

Since $a_s \sim \pi s$ when $s \to \infty$, it is not difficult to establish that the first series in the expression for w_1 converges to a continuous function for all $0 \leqslant y \leqslant h$ and $|x| < \infty$, with the exception of the neighbourhood of the point $x = 0$, where it behaves, apart from a factor, as $-\ln|x|$. As regards the second series in w_1, taking account of the fact that $\beta_s \sim \pi s/q$ when $s \to \infty$, it can be shown that for all $0 \leqslant y < h$ and $|x| < \infty$ it converges to a continuous function. When $y = h$, this series chiefly comprises the function (3.5), where now $t = \pi|x|/(\delta_* H)$. Thus, whey $y = h$, the seocnd series in w_1 has discontinuities of the first kind of the sgn type at the points $|x| = (2r + 2)\delta_* H (r = 0, 1, \ldots)$ and a break at the point $x = 0$.

It is interesting to note that, for the two different cases ($\upsilon > c_1$, $\upsilon < c_2$ and $\upsilon < c_1$, $\upsilon > c_2$), the behaviour of w_1 when $y = h$ is the same.

The first and second series in the expression for w_2 converge to continuous functions for all $-H \leqslant y \leqslant 0$ and $|x| < \infty$.

5. THE CASE OF AN SPR IN THE UPPER AND LOWER LAYERS

Suppose $\upsilon > c_1$ and $\upsilon > c_2$, where both Eqs (1.1) are equations of hyperbolic type. We will present the solution of Eqs (1.1), with $\gamma = i\gamma_*$ and $\delta = i\delta_*$, in the d'Alembert form

$$w_1 = f_1(x - \gamma_* y) + g_1(x + \gamma_* y), \quad w_2 = f_2(x - \delta_* y) + g_2(x + \delta_* y) \tag{5.1}$$

To determine the functions occurring in (5.1), we will use the method [3] of successive calculation for motion along the x axis in the negative direction from $x = 0$ of the incident, reflected and refracted shock wavefronts, employing a linear combination of the discontinuous function

$$\Pi(x) = \begin{cases} 0 & (x > 0) \\ -1 & (x < 0) \end{cases} \tag{5.2}$$

of different arguments. Note that, in this case, account is taken of the fact that there is no motion of points in the two-layer base when $x > 0$.

Further, without loss of generality in the reasoning, for simplicity we will assume that $h = H$.

Suppose $c_1 > c_2$; then the angle of refraction of the shock wavefronts in the lower layer will be greater than their angle of incidence in the upper layer (Fig. 2), and when $x > -4\gamma_* h$ the functions f_1, f_2, g_1 and g_2 in (5.1) must be taken in the form

$$\begin{aligned} &f_1(x - \gamma_* y) = A_1 \Pi[x - \gamma_*(y - h)] + A_2 \Pi[x - \gamma_*(y - 3h)] \\ &g_1(x + \gamma_* y) = B_1 \Pi[x + \gamma_*(y + h)] + B_2 \Pi[x + \gamma_*(y + 3h)] \\ &f_2(x - \delta_* y) = C_1 \Pi(x - \delta_* y + \gamma_* h) + C_2 \Pi(x - \delta_* y + 3\gamma_* h) \\ &g_2(x + \delta_* y) = D_1 \Pi[x + \delta_*(y + 2h) + \gamma_* h] \end{aligned} \tag{5.3}$$

Assuming that $3\gamma_*/2 < \delta_* < 2\gamma_*$, and then determining the coefficients $A_1, A_2, B_1, B_2, C_1, C_2$ and D_1 in (5.3) from boundary conditions (1.3), the last of which can be written in the form

$$\gamma_* G_1[-f_1(x - \gamma_* h) + g_1(x + \gamma_* h)] = T\Pi(x) \tag{5.4}$$

we finally obtain

$$\begin{aligned} w_1 = &-\frac{T}{G_1\gamma_*}\left\{\Pi[x - \gamma_*(y - h)] + \frac{1-k_*}{1+k_*}\Pi[x - \gamma_*(y - 3h)] + \right. \\ &\left. + \frac{1-k_*}{1+k_*}\Pi[x + \gamma_*(y + h)] + \left(\frac{1-k_*}{1+k_*}\right)^2 \Pi[x + \gamma_*(y + 3h)]\right\} \\ w_2 = &-\frac{T}{G_1\gamma_*}\left\{\frac{2}{1+k_*}\Pi(x - \delta_* y + \gamma_* h) - \frac{2}{1+k_*}\Pi[x + \delta_*(y + 2h) + \gamma_* h] + \right. \\ &\left. + \frac{2(1-k_*)}{(1+k_*)^2}\Pi(x - \delta_* y + 3\gamma_* h)\right\} \quad \left(k_* = \frac{G_2\delta_*}{G_1\gamma_*}\right) \end{aligned} \tag{5.5}$$

Now suppose $c_1 < c_2$; then the angle of refraction of the shock wave fronts in the lower layer is less than their angle of incidence in the upper layer (Fig. 3), and when $x > -3\gamma_* h$ the functions f_1 and g_2 in (5.1) must be taken in the form (5.3), while the functions g_1 and f_2 must be taken in the form

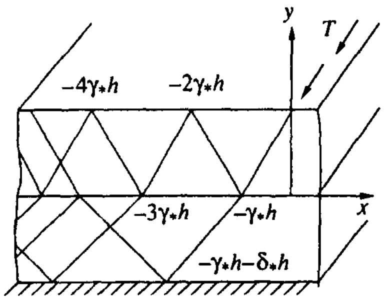

Fig. 2

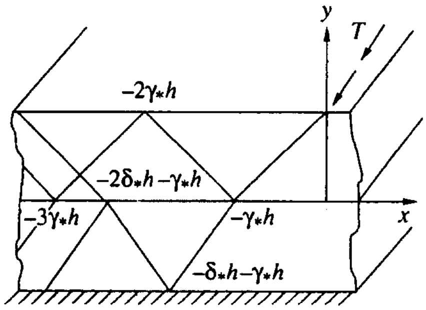

Fig. 3

$$g_1(x+\gamma_* y) = B_1\Pi[x+\gamma_*(y+h)] + B_2\Pi[x+\gamma_*(y+h)+2\delta_* h]$$
$$f_2(x-\delta_* y) = C_1\Pi(x-\delta_* y+\gamma_* h) + C_2\Pi[x-\delta_*(y-2h)+\gamma_* h] \tag{5.6}$$

Assuming that $\gamma_* > \delta_* > 2\gamma_*/3$, and then determining from boundary conditions (1.3) the coefficients $A_1, A_2, B_1, B_2, C_1, C_2$ and D_1, we will find, for w_1 and w_2, relations that differ from (5.5) by having the final term in brackets replaced respectively by

$$-\frac{4k_*}{(1+k_*)^2}\Pi[x+\gamma_*(y+h)+2\delta_* h], \quad \frac{2(1-k_*)}{(1+k_*)^2}\Pi[x-\delta_*(y-2h)+\gamma_* h] \tag{5.7}$$

In conclusion, we note that, in all cases of the relation of υ to c_1 and c_2, the change to an examination of the antiplane problem of the motion over a two-layer base of a shear load distributed along x is obvious in view of the superposition principle. In this case, all the discontinuities in the displacements will disappear, but similar discontinuities will remain in the stresses, including infinite discontinuities of the logarithmic type in the case when $\upsilon > c_1$ and $\upsilon < c_2$ inside the upper layer. This fact is of interest from the viewpoint of seismology problems.

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